The State-of-the-Art Parabolic Equation Approximation as Applied to Underwater Acoustic Propagation With Discussions on Intensive Computations

An Invited Paper Presented at the
108th Meeting of the Acoustical Society of America,
8-12 October 1984, Minneapolis, Minnesota

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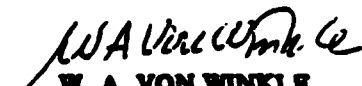86 10 1 115

PREFACE

This document was prepared under NUSC Project No. A65020, "Finite-Difference Solutions to Acoustic Wave Propagation," Principal Investigator, Dr. D. Lee (Code 3332). The sponsoring activity was the Naval Material Command, Program Manager, CAPT D. F. Parrish (NAVMAT 08L), Program Element 61152N, Navy Subproject/Task ZR00000101, Inhouse Laboratory Independent Research.

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This document contains the slide presentation entitled "The State-of-
the-Art Parabolic Equation Approximation as Applied to Underwater Acoustic
Propagation With Discussions on Intensive Computations," given at the
108th meeting of the Acoustical Society of America, 8-12 October 1984, in
Minneapolis, Minnesota.

The Parabolic Equation (PE) has applications in many different
scientific fields such as electromagnetics, optics theory, quantum

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20. Continued:

mechanics, plasma physics, seismology, underwater acoustics, and others.
The subject of this presentation is PE approximation as applied to underwater
acoustic wave propagation. A review will be given on past contributions, re-
cent developments will be highlighted, and, looking ahead, we will discuss
what the PE method can do in order to stimulate future research and develop-
ment, as well as applications. Intensive computations with respect to the PE
implementation will also be discussed.

THE STATE-OF-THE-ART PARABOLIC EQUATION APPROXIMATION
AS APPLIED TO UNDERWATER ACOUSTIC PROPAGATION WITH
DISCUSSIONS ON INTENSIVE COMPUTATIONS

The Parabolic Equation (PE) approximation was first introduced by Tappert [1,2,3] over a decade ago. Tappert's three papers, which are outlined in vugraph 1, are often referenced.

Computer Simulation of Long-Range Ocean Acoustic Propagation Using the Parabolic Equation Method

F. D. Tappert and R. H. Hardin

Eighth International Congress on Acoustics, p. 452, London (1974)
_____

The Parabolic Equation Approximation Method in Wave Propagation & Underwater Acoustics

F. D. Tappert
ed. J. B. Keller and J. S. Papadakis

Lecture Notes in Physics, Springer-Verlag (1977)
_____

Applications of the Split-Step Fourier Method to the Numerical Solution of Nonlinear and Variable Coefficient Wave Equation

R. H. Hardin and F. D. Tappert

SIAM Review 15, p.243 (1973)

VUGRAPH 1

In recent years, many improvements were made on the PE technique with respect to approximation, implementation, and application. Let us begin this discussion by a brief review of how the parabolic wave equation was derived by Tappert.

Consider the two-dimensional Helmholtz equation in cylindrical coordinates, that is,

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} + k_o^2 \, n^2(r,z) \, \phi = 0. \tag{1}$$

where $\phi(r,z)$ is the wave field, k_o is the reference wavenumber, $n(r,z)$ is the index of refraction, r indicates the range direction, and z indicates the depth direction.

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The PE approximation begins with the expression

$$\phi(r,z) = u(r,z)\, v(r),\qquad(2)$$

where $v(r)$ is strongly dependent on the range variable r while $u(r,z)$ is weakly dependent on r.

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{\partial^2\phi}{\partial z^2} + k_o^2\, n^2(r,z)\,\phi = 0$$

$$\phi(r,z) = u(r,z)\, v(r)$$

$$\left[v_{rr} + \frac{1}{r}v_r\right]u + \left[u_{rr} + \left(\frac{1}{r} + \frac{2}{v}v_r\right)u_r + u_{zz} + k^2 n^2(r,z)\right]v = 0$$

$$v_{rr} + \frac{1}{r}v_r + k_o^2 v = 0$$

$$u_{rr} + \left(\frac{1}{r} + \frac{2}{v}v_r\right)u_r + u_{zz} + k_o^2\left(n^2(r,z) - 1\right)u = 0$$

- -

$$v(r) = H_o^{(1)}(k_o r) \approx \sqrt{\frac{2}{\pi k_o r}}\, e^{i(k_o r - \frac{\pi}{4})}$$

$$u_{rr} + (2ik_o)\, u_r + u_{zz} + k_o^2\left(n^2(r,z) - 1\right)u = 0$$

- -

$$u_r = \frac{i}{2}\, k_o(n^2(r,z) - 1)u + \frac{i}{2k_o}\, u_{zz}$$

Standard PE

VUGRAPH 2

Substituting Eq. (2) into Eq. (1) gives

$$\left[v_{rr} + \frac{1}{r}v_r\right]u + \left[u_{rr} + \left(\frac{1}{r} + \frac{2}{v}v_r\right)u_r + u_{zz} + k_o^2\, n^2(r,z)\right]v = 0.\qquad(3)$$

Set the first term of Eq. (3) in brackets equal to $-k_o^2 v$ and the second term in brackets equal to $k_o^2 u$, we obtain two equations:

$$v_{rr} + \frac{1}{r}v_r + k_o^2 v = 0,\qquad(4)$$

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and

$$u_{rr} + \left(\frac{1}{r} + \frac{2}{v} v_r\right) u_r + u_{zz} + k_o^2 \left(n^2(r,z) - 1\right) u = 0. \tag{5}$$

Considering only the outgoing wave in the range direction, we see that the solution of Eq. (4) is the zeroth order Hankel function of the first kind, $H_o^{(1)}(k_o r)$.

Applying the farfield approximation, $k_o r \gg 1$, to the argument of $H_o^{(1)}(k_o r)$, we find that

$$v(r) = H_o^{(1)}(k_o r) \approx \sqrt{\frac{2}{\pi k_o r}} e^{i\left(k_o r - \frac{\pi}{4}\right)}. \tag{6}$$

Using Eq. (6) to simplify the coefficient $(1/r + (2/v) v_r)$, in Eq. 5., we find

$$u_{rr} + (2ik_o) u_r + u_{zz} + k_o^2 \left(n^2(r,z) - 1\right) u = 0. \tag{7}$$

Dropping u_{rr}, based on the paraxial approximation, $|u_{rr}| \ll |2ik_o u_r|$, we find

$$u_r = \frac{i}{2} k_o \left(n^2(r,z) - 1\right) u + \frac{i}{2k_o} u_{zz}. \tag{8}$$

This is the first parabolic wave equation derived by Tappert and is referred to as the STANDARD PE.

I would like to take another approach to Eq. (8), which you will find useful in observing some important physical properties.

$$u_{rr} + 2ik_0 u_r + u_{zz} + k_0^2(n^2(r,z) - 1)u = 0$$

$$\left(\frac{\partial}{\partial r} + ik_0 - ik_0 \sqrt{1 + (n^2 - 1) + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}}\right)\left(\frac{\partial}{\partial r} + ik_0 + ik_0 \sqrt{1 + (n^2 - 1) + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}}\right)u = 0$$

[NO SCATTERING]

One-way outgoing wave: $\left(\frac{\partial}{\partial r} + ik_0 - ik_0 \sqrt{1 + (n^2 - 1) + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}}\right)u = 0$

$$\sqrt{1 + (n^2 - 1) + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}} = 1 + \frac{1}{2}\left[(n^2 - 1) + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}\right]$$

$$u_r = \frac{i}{2}k_0(n^2(r,z) - 1)u + \frac{1}{2k_0}u_{zz}$$

VUGRAPH 3

If we neglect the scattering effects, Eq. (7) can be expressed in an operator form

$$\left(\frac{\partial}{\partial r} + ik_0 - ik_0 \sqrt{1 + (n^2 - 1) + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}}\right)\left(\frac{\partial}{\partial r} + ik_0 + ik_0 \sqrt{1 + (n^2 - 1) + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}}\right)u = 0. \quad (9)$$

Note that Eqs. (7) and (9) are the same if and only if there is no scattering.

Again, considering only the one-way outgoing wave, we deal with the solution

$$\left(\frac{\partial}{\partial r} + ik_0 - ik_0 \sqrt{1 + (n^2 - 1) + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}}\right)u = 0. \quad (10)$$

Making use of the slowly varying property of n(r,z) upon r, for the time being, we approximate the square-root operator by

$$\sqrt{1 + (n^2 - 1) + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}} \approx 1 + \frac{1}{2}\left[(n^2 - 1) + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}\right].$$

(11)

We refer to this as the small angle approximation. Substituting Eq. (11) into Eq. (10), we obtain the standard PE, that is,

$$u_r = \frac{i}{2}k_0\left(n^2(r,z) - 1\right)u + \frac{i}{2k_0}u_{zz}.$$

The derivation is left for the audience. It is important to note that at this point the standard PE, based on the PE approximation, obeys the following limitations:

LIMITATIONS

1. Farfield Approximation, $k_0 r \gg 1$.

2. $n(r,z)$ Slowly Varying in r.

3. One-Way Outgoing Wave.

4. No Scattering.

5. A Particular Square-Root Approximation.

VUGRAPH 4

Within these limitations, the standard PE is a very good mathematical model for long range, low frequency propagation. At this stage, the only effective solution algorithm for the standard PE was the split-step Fourier algorithm by Tappert and Hardin.[3]

Before I mention some earlier important developments, I want to mention the "Workshop on Wave Propagation and Underwater Acoustics," held in Mystic, Connecticut, in November 1974. This workshop was not limited to PE, but, after attending this workshop, a comprehensive article on PE was written by Frederick D. Tappert ("The Parabolic Approximation Method,"[2]), which is a chapter of the book cited in vugraph 5.

5

The Parabolic Equation Approximation in
Wave Propagation & Underwater Acoustics

F. D. Tappert
ed. J. B. Keller and J. S. Papadakis

Lecture Notes in Physics, Springer-Verlag (1977)

VUGRAPH 5

To this date, the Tappert article is still the most comprehensive article on PE.

In 1977, there was another workshop[4] in Woods Hole, Massachusetts, where Tappert gave another paper on the application of his split-step algorithm. Not many people are familiar with this paper.

Selected Applications of the Parabolic Equation Method
in Underwater Acoustics

F. D. Tappert

International Workshop on Low-Frequency Propagation
& Noise, Vol. 2, Woods Hole, Mass. pp. 155-194,(1974).

VUGRAPH 6

Since 1974, the interest in the PE was on the rise, but little was done. However, there were a number of notable contributions.

The first practical results of applying the PE were published by C. W. Spofford.[5] Then a few papers [6-12] were published in relation to the normal mode method and its application. These are

A Synopsis of the AESD Workshop on Acoustic-Propagation
Modeling by Non-Ray-Tracing Techniques, C. W. Spofford,
AESD TN-73-05 (1973) _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _

Eikonal Approximation and the Parabolic Equation, D. R.
Palmer, J. Acoust. Soc. Am., 60(2), 343-354 (1976) _ _ _ _ _

Relation Between the Solutions of the Helmholtz and
Parabolic Equation for Sound Propagation, J. A. DeSanto, J.
Acoust. Soc. Am., 62(2), 295-297 (1977) _ _ _ _ _ _ _ _

A Correction to the Parabolic Equation, J. A. DeSanto, J. S.
Perkins, and R. B. Baer, J. Acoust. Soc. Am., 64(6), 1664-1666
(1978) _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _

On the Parabolic Approximation to the Reduced Wave
Equation, G. A. Kriegsmann & E. W. Larsen, SIAM J. Appl.
Math., 34(1), 201-204 (1978) _ _ _ _ _ _ _ _ _ _

Helmholtz Equation as an Initial Value Problem with
Approximation to Acoustic Propagation, R. M. Fitzgerald,
J. Acoust. Soc. Am. 57(4), 839-842, (1975) _ _ _ _ _ _ _ _

Propagation of Normal Mode in the Parabolic
Approximation, S. T. McDaniel, J. Acoust. Soc. Am., 57(2),
307-311 (1975) _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _

Parabolic Approximation for Underwater Sound Propagation,
S. T. McDaniel, J. Acoust. Soc. Am., 58(6), 1178-1185 (1975)

VUGRAPH 7

Researchers are curious and interested in the relationship between the
PE and the Helmholtz equation. Filtzgerald[10] analyzed the PE in terms of
normal mode theory. Dave Palmer [6] observed that the normal mode formulism
was difficult in ordering the geometric optics path-length parameter because
of mode-coupling. The PE removes this difficulty.

Then, during approximately the same time period, a few interesting
developments happened, one was the computer code.

7

The Use of the Parabolic Equation Method in Sound
Propagation Modeling,

F. B. Jensen and H. Krol

SACLANTCEN MEMO SM-72 (1975)

The AESD Parabolic Equation Model,

H. K. Brock

NORDA TN-12 (1978)

VUGRAPH 8

Jensen[13] had a PE package called PAREQ in his laboratory performing
a variety of research and applications; so too did Brock.[14] These computer
models used the split-step algorithm to solve the standard PE. G.
Gartrell[15] wrote a split-step code on the IBM 370/168.

However, in solving the standard PE, a number of users found a phase
error. DeSanto, Perkins, and Baer[8] discussed this phase error and
introduced a correction to the parabolic approximation. Notable was a
technique introduced by Brock, Buchal, and Spofford[16] to modify the
sound-speed profile to improve the accuracy of the PE.

Modifying the Sound-Speed Profile to Improve the Accuracy of
the Parabolic Equation Technique

H. K. Brock, R. N. Buchal, and C. W. Spofford

J. Acoust. Soc. Am., 62(3), pp. 543-552 (1977)

VUGRAPH 9

Interest was also increasing in the application of the PE to solve
real problems. The PE models in various laboratories were all based on the
use of the split-step Fourier algorithm with an artificial bottom treatment.

In order to apply the PE to solve real problems, the model would be
required to have many capabilities. The natural question is what can PE do?

Can PE offer these capabilities. These questions stirred up research and development interest. I ask the question in a different way -- what can we do to improve the PE capability?

First, to improve the PE capability, Lee, Papadakis, and Prieser[17,18] initiated the numerical solution of the parabolic wave equation so that under shallow water or strong bottom interaction environments the numerical technique can handle the bottom boundary condition.

Numerical Solution of the Parabolic Wave Equation:
An Ordinary-Differential-Equation Approach

D. Lee and J. S. Papadakis

J. Acoust. Soc. Am., 68, pp. 1482-1488 (1980)

Generalized Adams Methods for Solving Underwater Wave
Propagation Problems

D. Lee and S. Preiser

J. Comp. & Math. with Appls., 7(2), pp. 195-202 (1981)

Finite-Difference Solution to the Parabolic Wave Equation

D. Lee, G. Botseas, and J. S. Papadakis

J. Acoust. Soc. Am., 70(3), pp. 795-800 (1981)

VUGRAPH 10

Lee and Papadakis introduced the approach of using an ordinary differential equation (ODE) to determine a special kind of bottom boundary (rigid) within the framework of the PE.

This bottom boundary treatment was incorporated into the ODE and finite difference models. At that time, we did not have a better implementation of the ODE solution, but we concentrated our efforts on developing a very basic, general purpose finite-difference scheme,[19] which could be implemented into computer code.[20] This scheme is known today as the Implicit Finite Difference (IFD) model and is an implicit Crank-Nicolson

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method, which is unconditionally stable. The IFD model is used quite often to solve the standard PE.

We used the ODE model to solve a wedge problem consisting of range versus propagation loss. The results turned out surprisingly well.

Shallow-to-Deep Water Propagation   Wedge Solution Comparison
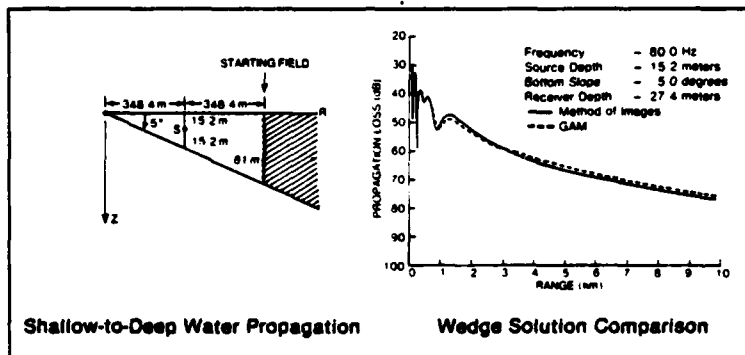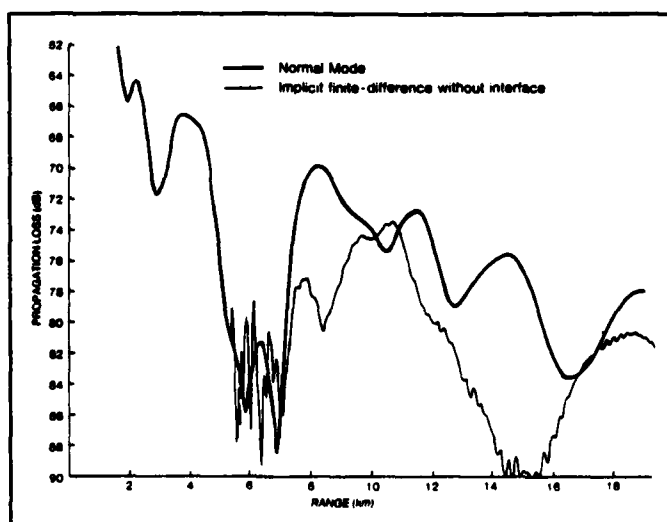
VUGRAPH 11

After we showed that our bottom boundary treatment worked, we were asked many questions pertaining to our IFD model's capabilities. One of the first practical questions was a problem proposed by H. P. Bucker. He asked: Your IFD model can handle bottom boundary conditions -- can it handle the interface condition?

The problem did not seem to be a particularly difficult one, however, at that time, no existing PE code could do it.

Bucker Problem

Sound Speed Profile
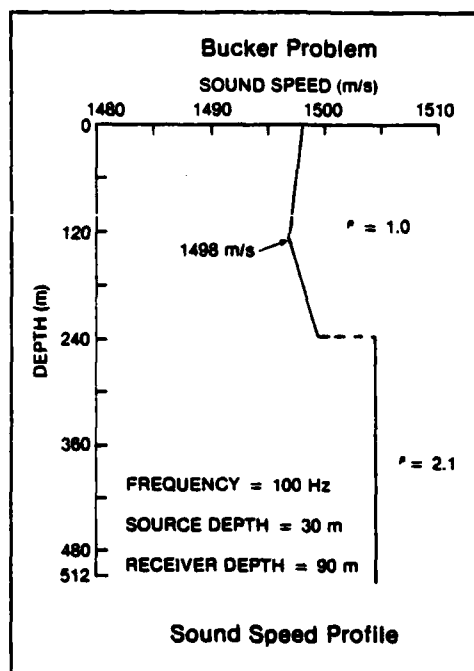
VUGRAPH 12

The problem was solved easily by the normal mode solution. The normal mode solution was used as a reference solution for comparison of transmission loss predictions.

VUGRAPH 13

11

Handling the interface wasn't difficult, but it required some thought as to the best approach. Thanks to many valuable discussions with Dr. Suzanne T. McDaniel, we worked out a finite-difference treatment for the horizontal interface.[21] The horizontal interface development is documented in the article outlined in vugraph 14.
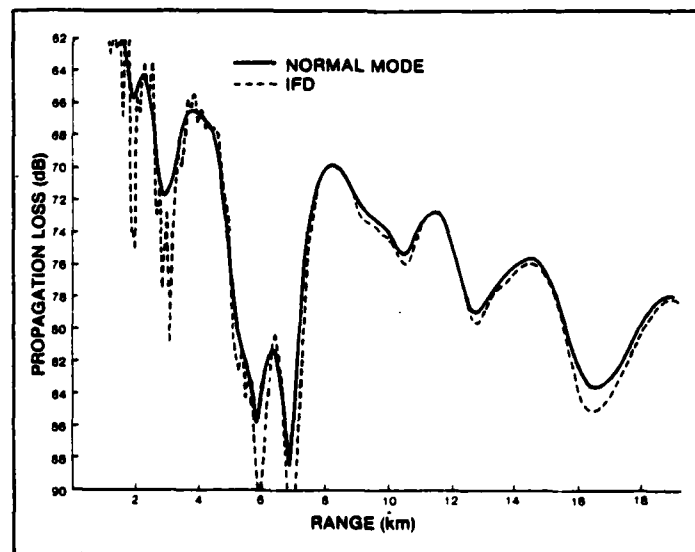
A Finite-Difference Treatment of Interface Conditions for the Parabolic Wave Equation: The Horizontal Interface

S. T. McDaniel and Ding Lee

J. Acoust. Soc. Am., 71(4), pp. 855-858 (1982)

VUGRAPH 14

We then incorporated the horizontal interface conditions into the finite difference code and ran the Bucker problem on the VAX 11/780 computer at NUSC.

VUGRAPH 15

As vugraph 15 shows, without the interface treatment, the results from both IFD and split-step do not agree with our normal mode reference solution results, but after the finite-difference treatment, the results are in reasonable agreement.

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Naturally, it is logical to extend the finite-difference technique to handle the irregular interface condition.[22] Dr. McDaniel and I have worked this out and I shall talk about this a little later.

While the capabilities of the PE were being developed, use of the PE model was increasing. During this period of development, NORDA sponsored a workshop, the "NORDA Parabolic Equation Workshop."[23] Many interesting, realistic problems were introduced at the workshop and a comprehensive report was published.
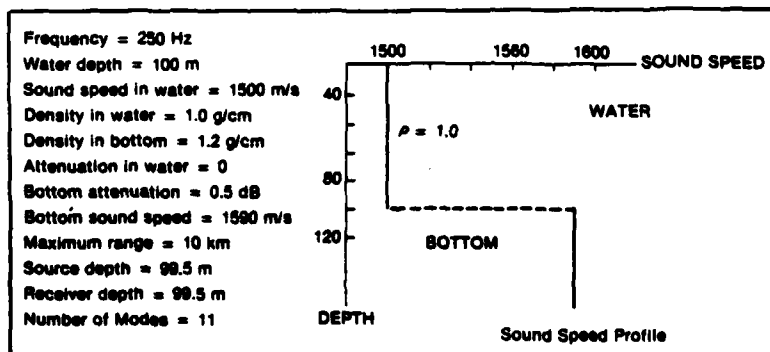
NORDA Parabolic Equation Workshop

James A. Davis, DeWayne White, and Raymond C. Cavanagh

NORDA TN-143 (1981)

VUGRAPH 16

One of the problems introduced was that of wide angle propagation.

Frequency = 250 Hz
Water depth = 100 m
Sound speed in water = 1500 m/s
Density in water = 1.0 g/cm
Density in bottom = 1.2 g/cm
Attenuation in water = 0
Bottom attenuation = 0.5 dB
Bottom sound speed = 1590 m/s
Maximum range = 10 km
Source depth = 99.5 m
Receiver depth = 99.5 m
Number of Modes = 11

Sound Speed Profile

VUGRAPH 17

This problem was solved satisfactorily by both the normal mode and the fast field program (FFP). We then used the FFP solution as a benchmark reference solution.
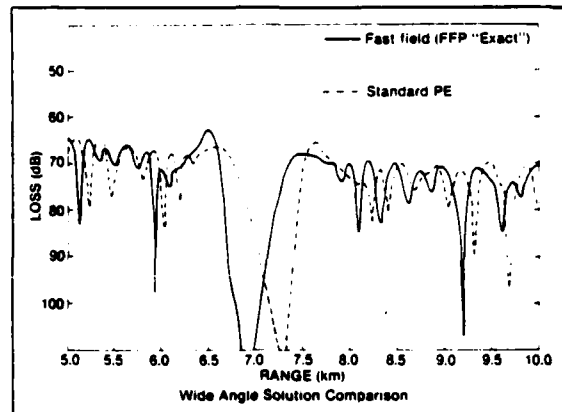
VUGRAPH 18

Note, in vugraph 18, that the dashed-dotted line was the solution produced by the standard PE (both the split-step and the IFD), the disagreement is clear. It was due to the size of the angle of propagation. An important capability required to produce agreement is the wide angle capability.

Let me briefly describe the mathematical development of the wide angle capability. Recall the one-way outgoing wave equation, that is,

$$\left(\frac{\partial}{\partial r} + ik_0 - ik_0 \sqrt{1 + (n^2 - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2}}\right) u = 0 ,$$

where we made the approximation

$$\sqrt{1 + (n^2 - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2}} \approx 1 + \frac{1}{2}\left[(n^2 - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2}\right]$$

to obtain the standard PE

$$u_r = \frac{i}{2} k_0 \left(n^2(r,z) - 1\right) u + \frac{i}{2k_0} u_{zz} .$$

One-way outgoing wave: $\left(\frac{\partial}{\partial r} + ik_0 - ik_0 \sqrt{1 + (n^2 - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2}}\right) u = 0$

$$\sqrt{1 + (n^2 - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2}} \approx 1 + \frac{1}{2}\left[(n^2 - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2}\right]$$

$$u_r = \frac{i}{2} k_0 (n^2(r,z) - 1) u + \frac{i}{2k_0} u_{zz}$$

VUGRAPH 19

14

To be general, we approximate the square-root operator by a rational function approximation as follows:

$$\sqrt{1 + \left(n^2(r,z) - 1\right) + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}} \approx \frac{1 + p\left[\left(n^2(r,z) - 1\right) + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}\right]}{1 + q\left[\left(n^2(r,z) - 1\right) + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}\right]} . \qquad (12)$$

If $p = 1/2$ and $q = 0$, we see clearly it reduces to the small angle approximation, thus, resulting in the standard PE.

For a special selection, $p = 3/4$ and $q = 1/4$, the right-hand side of Eq. (12) becomes

$$\sqrt{1 + \left(n^2(r,z) - 1\right) + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}} \approx \frac{1 + \frac{3}{4}\left[\left(n^2(r,z) - 1\right) + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}\right]}{1 + \frac{1}{4}\left[\left(n^2(r,z) - 1\right) + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}\right]} , \qquad (13)$$

which is the rational function approximation of the square-root operator by John F. Claerbout.[25] We choose to keep p and q arbitrary so that we can determine p and q to suit our needs.

If we use Eq. (12) for the square-root operator and substitute it into the one-way outgoing wave equation, we find a pseudopartial differential equation, that is,

$$\frac{\partial}{\partial r} u = \left(-ik_0 + ik_0 \frac{1 + p\left[\left(n^2(r,z) - 1\right) + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}\right]}{1 + q\left[\left(n^2(r,z) - 1\right) + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}\right]}\right) u , \qquad (14)$$

which is the Wide Angle Wave Equation.

A Rational Function Approximation

$$\sqrt{1 + \left(n^2(r,z) - 1\right) + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}} \approx \frac{1 + p\left[(n^2(r,z) - 1) + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}\right]}{1 + q\left[(n^2(r,z) - 1) + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}\right]}$$

$$\sqrt{1 + (n^2(r,z) - 1) + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}} \approx \frac{1 + \frac{3}{4}\left[(n^2(r,z) - 1) + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}\right]}{1 + \frac{1}{4}\left[(n^2(r,z) - 1) + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}\right]}$$

Jon F. Claerbout approximation

Wide Angle Wave Equation

$$\frac{\partial}{\partial r} u = \left(-ik_0 + ik_0 \frac{1 + p\left[(n^2(r,z) - 1) + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}\right]}{1 + q\left[(n^2(r,z) - 1) + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}\right]}\right) u$$

VUGRAPH 20

We tend to refer to it as the Wide Angle PE, but in a sense this is deceptive. The PE is our key equation. When $p = 1/2$ and $q = 0$, the standard PE is a special case. Therefore, when we refer to the Wide Angle PE, we are actually referring to the pseudopartial differential equation (14).

With the wide angle capability, you see that the IFD produced agreeable results when compared with the benchmark FFP solution.

Wide Angle Solution Comparison

VUGRAPH 21

16
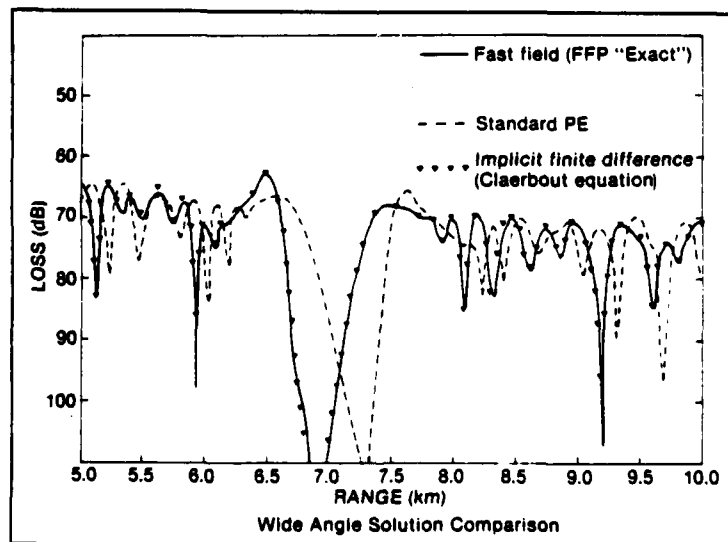
There have been a number of authors[24-27] who have contributed to the theoretical development of the wide angle capability.

Fundamentals of Geophysical Data Processing with Applications to Petroleum Prospecting

Jon F. Claerbout

McGraw-Hill (1976)

High Angle PE

Robert R. Greene

NORDA Parabolic Equation Workshop (1981)

IFD: Wide Angle Capability

George Botseas, Ding Lee, and Kenneth E. Gilbert

NUSC TR#6905 (1983)

Extension of the Parabolic Equation Model for High-Angle Bottom-Interacting Paths

L. B. Dozier and C. W. Spofford

SAI Technical Report SAI-78-712-WA (1977)

VUGRAPH 22

Greene used the term "rational parabolic" as opposed to "parabolic." He also applied the rational function approximation to the square-root operator. This rational function approximate technique was applied earlier by Claerbout.[25]

A number of other people also examined and developed the wide angle capability. At NUSC, we incorporated this into our IFD code and a comprehensive report was published.[24] In the seismology field, Berkhout used the continued fraction to approximate the square root operator.[28]

Wave Field Extrapolation Technique in Seismic Migration

A. J. Berkhout

Geophysics, Vol. 46, #12, pp. 1838-1656 (1981)

VUGRAPH 23

It is interesting to note that Berkhout's first order continued fraction
coincides exactly with Tappert's standard small angle PE. His second order
continued fraction coincides with the Claerbout approximation. Roughly
speaking, the small angle PE can accommodate propagation angles up to 15°,
and wide angle PE (IFD) can accommodate propagation angles up to 40°.

David Thomson[29] was the first to apply the Split-step algorithm to
handle the wide angle.

A Wide Angle Split-Step Algorithm for the Parabolic
Equation

David J. Thomson and N. R. Chapman

J. Acoust. Soc. Am., 74(6), pp. 1848-1854 (1983)

VUGRAPH 24

Recently, one of my colleagues, Donald St. Mary[30] of the University of
Massachusetts, developed a very high angle PE using a higher order rational
approximation that can accommodate a propagation angle greater than 45°.
This is good news in the area of shear wave propagation.

Formulation and Discretization of a Very Wide Angle
Parabolic Equation

Donald F. St. Mary and Ding Lee (1984)

VUGRAPH 25

All the authors I have mentioned thus far have made contributions to
the wide angle capability in this period of time. Two researchers, Estes and
Fain,[31] started earlier examining the wide angle formulation using the
Taylor series expansion.

Numerical Technique for Computing the Wide Angle
Acoustic Field in an Ocean with Range-Dependant
Velocity Profiles

L. E. Estes and G. Fain

J. Acoust. Soc. Am., 62(1), pp. 38-43 (1977)

VUGRAPH 26

Continuous use of the PE for research and application purposes showed
reasonable success, however, under unusual environments, it was not fully
developed to handle everything. SACLANT and NORDA called our attention to
the fact that in the irregular sloping interface situation, if the depth
partition points do not fall on the interface boundary, inaccuracy will
occur. In numerical analysis language, this is called numerical reflections.
The present computer code cannot handle the situation without some
modification because the irregular interface conditions treatment is not
included in the present code. In addition to the progress made by McDaniel
and me, recent progress has been made by Jaeger[32] to treat the interface
condition by using the irregular interface condition developed by McDaniel
and me.

A Computer Program for Solving the Parabolic Equation
Using an Implicit Finite-Difference Solution Method
Incorporating Exact Interface Conditions

Larry Ernest Jaeger

Naval Postgraduate School, MS Thesis (1983)
──
A Finite-Difference Treatment of Interface Conditions for the
Parabolic Wave Equation: The Irregular Interface

Ding Lee and S. T. McDaniel

J. Acoust. Soc. Am., 73(5), pp. 1441-1447 (1983)

VUGRAPH 27

Independently from the direct application of the irregular interface condition, Jules deG Gribble[33] made an important improvement on the finite difference model.

Jules allows variable mesh spacing and deals with interfaces that do not lie on a mesh point. He extended the finite difference treatment of interfaces to the parabolic wave equation. Jules' main effort was centered at the employment of variable vertical mesh size in conjunction with a high level system of ODE solver.

Extending the Finite Difference Treatment of Interfaces When Using the Parabolic Wave Equation

Jules deGribble

J. Acoust. Soc. Am., 76(1), pp. 217-221 (1984)

VUGRAPH 28

Up to this point, it seems that there is enough PE capability for research studies as well as for applications. Three types of PE models exist.

EXISTING PARABOLIC EQUATION MODELS

1. Split-Step Fourier Algorithm Model

2. Implicit Finite-Difference Model

3. Ordinary-Differential-Equation Model

VUGRAPH 29

Little has been done to compare the available computer models. The only published literature was given by Kewley[34] who discussed practical solutions of the PE model for underwater acoustic wave propagation.

Practical Solutions of the Parabolic Equation Model for
Underwater Acoustic Wave Propagation
in Computational Techniques & Applications Conference

D. J. Kewley, L. T. Sin Fai Lamn, and G. Gartrell
ed. J. Noye, Sydrey

North-Holland, Amsterdam (1983)

VUGRAPH 30

Among these three models, IFD is the more general purpose. The ODE
solution has great potential, but is not yet fully developed. Applications
of the PE model indicates that it is doing well as a research code,
especially the IFD code, because of its accurate computation of the wave
field. However, a number of users are actually using these codes for
applications.

In solving either the small angle PE or the wide angle wave equation,
we deal with one input parameter, the reference wavenumber k_o. Application
of the above equations to solve real problems requires a clever selection of
the reference wavenumber k_o. This selection was ignored by the model
developers and the users. For the user, in practice, he does not usually
have the knowledge to select the best k_o; therefore, users have, in most
cases, ignored the selection of k_o. The model developer built in the
automatic selection of k_o according to physical experiences. Inappropriate
k_o will lead to an evident phase error. Pierce recently reemphasized the
importance of k_o selection and introduced a formula to determine the range
of k_o based on the Rayleigh quotient. Some numerical experiments have been
carried out at the Naval Underwater Systems Center, New London Laboratory;
results show some phase shift effects dependent on k_o variations. Tappert
and Lee joined Pierce in studying the natural selection of k_o.

21

> The Natural Reference Wavenumber for Parabolic
> Approximations in Ocean Acoustics
>
> Allan D. Pierce
>
> J. Comp. & Math. with Appls. (To be published) (1984)

VUGRAPH 31

It is interesting to note that L. Ngheim-Phu and Tappert[36] developed a PE box that can be put on shipboard for fast prediction.

> A High-speed.Compact, and Interactive Parabolic Equation
> SOlution GENerator System (PESOGEN)
>
> Lan Ngheim-Phu and F. D. Tappert
>
> J. Acoust. Soc. Am., 75(S1), p.s26 (1984)

VUGRAPH 32

The interest in the PE solution has continuously increased. An interesting development was the Yale University Workshop.[37] Three days were devoted to discussions by invited speakers about the state-of-the-art of computational ocean acoustics.

> **COMPUTATIONAL OCEAN ACOUSTICS WORKSHOP**
>
> **YALE UNIVERSITY**
>
> Sponsors: ONR (Math. Dept.)
>   Yale U. (Computer Science Dept.)
>   NUSC (Independent Research)
>
> Special Issue: J. Comp. & Math. with Applications
>
> Book: Pergamon Press
>   Editors: Martin H. Schultz and Ding Lee

VUGRAPH 33

Besides the book published by the Pergamon Press, there was a technical
document outlining the progress in the development and application of the PE.

```
Recent Progress in the Development and Application of the
Parabolic Equation

Ed. Paul D. Scully-Power and Ding Lee

NUSC TD7145 (1984)
```

VUGRAPH 34

Most contributions we have mentioned so far deal with the solution of
linear PE. McDonald and Kuperman[38] developed a two-dimensional (range and
depth) formula for the propagation of nonlinear acoustic pulses and weak
shocks in a refracting medium. The equation they developed is the nonlinear
time domain counterpart of the linear frequency domain PE.

```
Time Domain Solution of the Parabolic Equation Including

Nonlinearity, E. B. McDonald and W. A. Kuperman, To appear

in J. Comp. & Math. with Appls. (1985)
```

VUGRAPH 35

This technical document[39] contains some of the interesting topics Tappert
and I discussed over a period of two summers. These include

- Range Refraction Corrected PE.(ref 40)
- Density Effects in PE.(ref 39)
- Time Domain PE.
- High Frequency PE (hybrid ray-PE). (ref 39)
- Backscattering (in range) PE.(ref 41)
- Rough Boundaries in PE.(ref 42)
- 3-Dimensional PDE.(ref 43)
- Stable Explicit Schemes for Solving 3-Dimensional (2-Dimensional) PE
- Shear Waves in PE.
- Match Interface Conditions Between Fluid and Elastic Media.
- Large Angle PE.
- Currents in PE.
- Doppler Effects in PE.
- Moving Sources and Receivers, or Time-Variables.
- The Application of Multi-Array Processors to Solve the Parabolic Wave Equation
- Reciprocity in the Time Domain and the PE Method.
    -
    -
    -

VUGRAPH 36

As for the computation of long range propagation, it is on a very large scale. Ideally, we would like to have in principle

LARGE SCALE COMPUTATIONS

1. Accuracy

2. Speed

3. Minimal computer storage requirements

VUGRAPH 37

Let me use the IFD as a simple example. Using the IFD to solve the parabolic wave equation, either small angle or wide angle, requires the solution of a system of equations of the form

$$Au^{n+1} = Bu^n + u^n + u^{n+1}$$

VUGRAPH 38

24

Since both matrices A and B are tridiagonal, a special recursive formula is
applied to solve the system at minimal cost. Not counting the overhead, the
tridiagonal solver requires 6N operations. The solution looks so simple and
economical that it does not seem to require intensive computations even for
long range propagation problems. However, this is not generally true. Let us
recall our earlier solution to the wedge problem of a rigid bottom boundary
condition. Our early solution to this problem was applying the Generalized
Adams ODE method, which produced the accurate solution.

Shallow-to-Deep Water Propagation   Wedge Solution Comparison
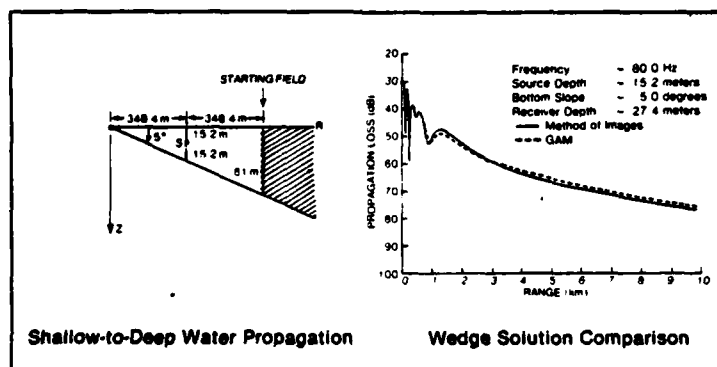
VUGRAPH 39

For the sloping bottom boundary, in order to solve the system
satisfactorily, we adopted a variable dimension procedure. We started at
approximately 348 m for a 0.15 m depth increment and ended up to solve an
initial system of equations of order 400. Due to the memory storage limit,
calculations were not allowed to go up to a maximum range of 10 km, which
means we have to solve a system of equations of order 5740. Recently, we
improved the technique and relaxed the storage requirement, thus, increased
the speed.

25

The parabolic wave equation can be solved by a first order Generalized Adams method.[44]

PE	$u_r = a(k_o,r,z)u + b(k_o,r,z)u_{zz}$
ODE	$u_r = A(k_o,r,z)u + g(k_o,r,z,u_o)$
GAB	$u^{m+1} = e^{Ah}u^n + h(Ah)^{-1}(e^{Ah}-I)g_n$
EXP	$e^{Ah} \approx (I-Ah)^{-1}$
Improved GAB	$(I-Ah)u^{n+1} = u^n + hg_n$

VUGRAPH 40

Since Matrix A increases dimension at every range step, e^{Ah} has to be updated at every range increment. It costs N^3 operations for the inversion and cost N^2 memory storages because we calculate the matrix exponential by the rational Pade approximation, which requires the inversion of an NxN matrix. Even though the Matrix A is tridiagonal, the inverse destroys the tridiagonal property and fills up the storage. You can see the cost. By a careful study, for a reasonable choice of the step size, we can calculate the new wave field by avoiding this expensive matrix exponential calculation and solving a system of equations.

This system is again tridiagonal, we reduce N^3 operations to 6N operations; moreover, we reduce a N^2 memory storage requirement to only 3N storages. By coincidence, this problem can also be solved nicely by the IFD. Now, one can again, by taking advantage of using hardware, improve the efficiency of the computation. You shall hear this from other talks.

CONCLUSIONS

Now you have heard of the many important contributions toward the application of PE approximation in solving ocean acoustic problems and have also heard of some interesting PE developments. You may note that there are many not yet well-developed PE approximations, many more than existing PE capabilities. However, I would like to call your attention to the fact that NOT every problem can be solved by the PE approximation in an efficient manner. If the physical conditions fall within the limitation of the PE approximation, PE approximation is an efficient method to apply.

> **NOT** every problem can be solved by
> the Parabolic Equation Approximation

VUGRAPH 41

I welcome your comments and participation in solving these problems and applying your solutions to realistic problems.

Thank you.

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